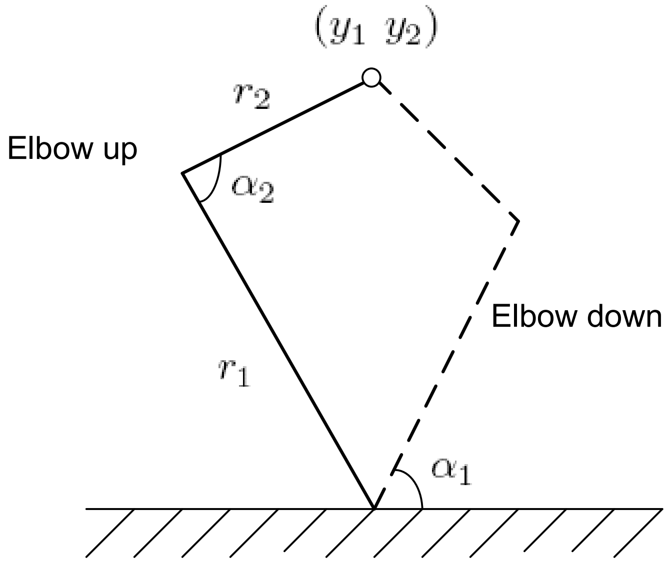


Fig. 1. Two link robot arm illustrating how the Cartesian coordinates (y_1, y_2) of the end effector is mapped to the given angles (α_1, α_2). The solid and dashed lines show the 'elbow up' and 'elbow down' situations, respectively.

I. Tracking a Robot Arm

This example illustrates the power of the Cubature Kalman Filter/Smoother (CKF/CKS). Fig. 1 shows two typical positions of a two-link robot arm, namely, 'elbow-up' and 'elbow-down'. Given the angles (α_1, α_2), the end effector position of the robot arm can be described in the Cartesian coordinate as follows:

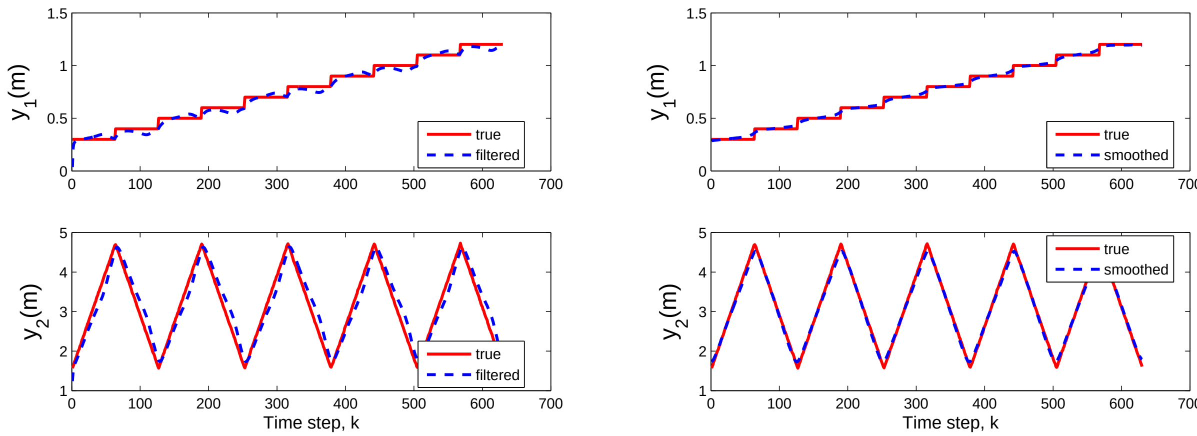
$$
\begin{aligned}
y_1 &= r_1 \cos(\alpha_1) - r_2 \cos(\alpha_1 + \alpha_2) \\
y_2 &= r_1 \sin(\alpha_1) - r_2 \sin(\alpha_1 + \alpha_2),
\end{aligned}
$$

where $r1 = 0.8$ and $r2 = 0.2$ are the lengths of the two links; $\alpha_1 \in [0.3, 1.2]$ and $\alpha_2 \in [\pi/2, 3\pi/2]$ are the joint angles confined to a specific region. The mapping from (α_1, α_2) to (y_1, y_2) is called the *forward kinematic*, whereas the *inverse kinematic* refers to the mapping from (y_1, y_2) to (α_1, α_2). The inverse kinematic is not a one-to-one mapping and thus its solution is not unique.

For the inverse kinematic problem, let the state vector $\mathbf{x}$ be $\mathbf{x} = [\alpha_1\ \alpha_2]^T$ and the measurement vector $\mathbf{y}$ be $\mathbf{y} = [y_1\ y_2]^T$. The state-space model of the problem is written as

$$
\begin{aligned}
\text{State equation:} \qquad \mathbf{x}_{k+1} &= \mathbf{x}_k + \mathbf{w}_k \\
\text{Measurement equation: } \mathbf{y}_k &= \begin{pmatrix} \cos(\alpha_{1,k}) & -\cos(\alpha_{1,k} + \alpha_{2,k}) \\ \sin(\alpha_{1,k}) & -\sin(\alpha_{1,k} + \alpha_{2,k}) \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \end{pmatrix} + \mathbf{v}_k
\end{aligned}
$$

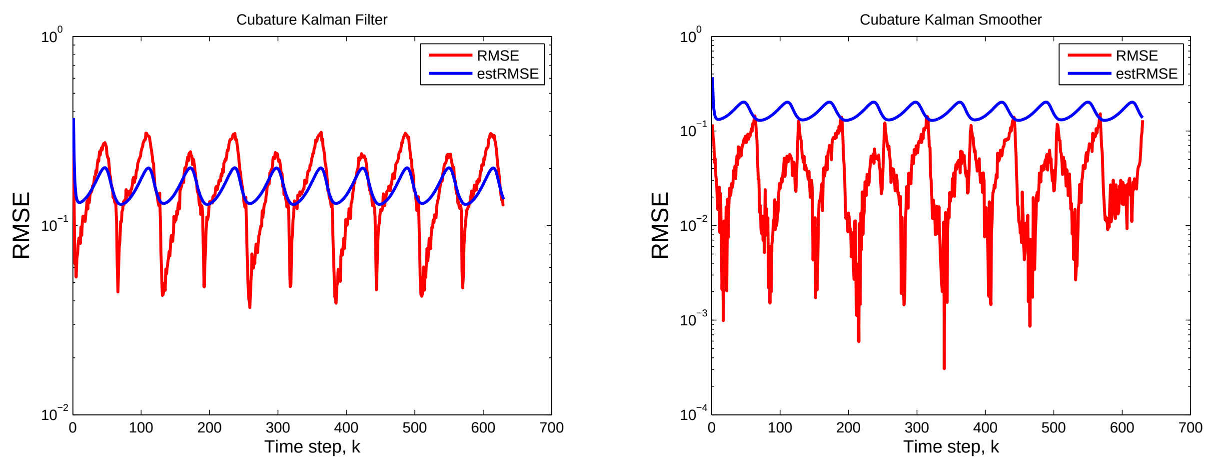
Here, we assume the state equation to follow a random-walk model perturbed by white Gaussian noise



(a) Cubature Kalman Filter (CKF)

(b) Cubature Kalman Smoother (CKS)

Fig. 2. Tracking results (True trajectory- Solid line, Estimated Trajectory- Dotted line)



(a) Cubature Kalman Filter (CKF)

(b) Cubature Kalman Smoother (CKS)

Fig. 3. Ensemble averaged (over 50 runs) root mean-squared error (RMSE) results (true rmse- red line, estimated rmse- blue)

$\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \mathrm{diag}[0.01, 0.1])$. The measurement equation is nonlinear with additive measurement noise $\mathbf{v} \in \mathcal{N}(0, 0.005\mathbf{I})$, where $\mathbf{I}$ is the two-dimensional identity matrix. As can be seen from Fig. 2, α_1 is a slowly increasing process with periodic random walk whereas α_2 is a periodic, fast, and linearly-increasing/decreasing process. As depicted by Figs. 3(a) and 3(b), the root mean squared error of the CKS is less than that of the CKF as expected.

PS: Please find more about Cubature Filtering at http://grads.ece.mcmaster.ca/ aienkaran/.