CKF-IMM FOR TARGET TRACKING

The interacting Multiple Model (IMM) estimator is well-known within the tracking community for its adaptive estimation capability (Section 11.7.4, *Estimation with Application to Tracking and Navigation* By Bar-Shalom *et al.*). The objective of this note is to integrate the square-root formulation of a relatively new nonlinear Bayesian estimator called the *Cubature Kalman Filter* (CKF) with the IMM. The resulting estimator is called the *CKF-IMM*.

In the air traffic control tracking scenario under consideration, starting from $[25,000m, 10,000m]$ at time $t = 0$ s, the aircraft flies eastward for 125 s at 120 m/s, before executing a $1°/s$ coordinated turn for 90 s. Then it flies northward for another 125 s, followed by a $3°/s$ turn for 30 s. After the turn, it continues to fly eastward at constant velocity. The aircraft motion can be therefore described by two different motion model segments- the (nearly) constant velocity model and the coordinated turn model. The nearly constant velocity model is described by the following linear equation:

$$\mathbf{x}_{k+1} = \begin{pmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{pmatrix} \mathbf{x}_k + \begin{pmatrix} \frac{1}{2}T^2 & 0 \\ T & 0 \\ 0 & \frac{1}{2}T^2 \\ 0 & T \end{pmatrix} \mathbf{v}_k, \quad (1)$$

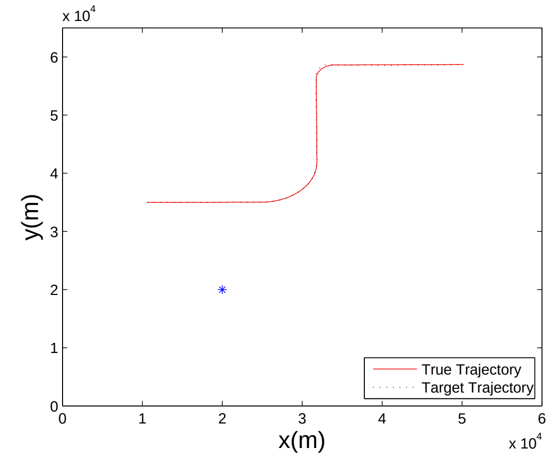
where T is the measurement sampling interval and $\mathbf{x}$ is the state of the aircraft consisting of the aircraft's position and velocity, both in the x- and y-directions, respectively. The nonlinear coordinated-turn model is given by

$$\mathbf{x}_{k+1} = \begin{pmatrix} 1 & \frac{\sin(\omega T)}{\omega} & 0 & \frac{-(1-\cos(\omega T))}{\omega} & 0 \\ 0 & \cos(\omega T) & 0 & -\sin(\omega T) & 0 \\ 0 & \frac{-(1-\cos(\omega T))}{\omega} & 1 & \frac{\sin(\omega T)}{\omega} & 0 \\ 0 & \sin(\omega T) & 0 & \cos(\omega T) & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \mathbf{x}_k + \begin{pmatrix} \frac{1}{2}T^2 & 0 & 0 \\ T & 0 & 0 \\ 0 & \frac{1}{2}T^2 & 0 \\ 0 & T & 0 \\ 0 & 0 & T \end{pmatrix} \mathbf{v}_k, \quad (2)$$

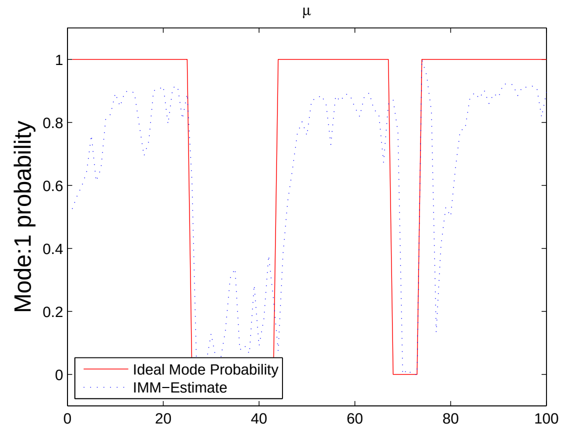
where the turn rate (ω) of the aircraft is represented by the fifth state variable.

The radar, stationed at [20,000 m, 20,000 m], is assumed to measure range and bearing measurements:

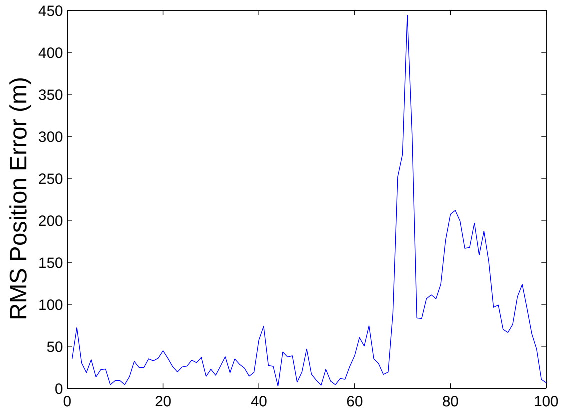
$$\mathbf{z}_{k+1} = \begin{pmatrix} \sqrt{\mathbf{x}_{1_{k+1}}^2 + \mathbf{x}_{3_{k+1}}^2} \\ \tan^{-1} \frac{\mathbf{x}_{3_{k+1}}}{\mathbf{x}_{1_{k+1}}} \end{pmatrix} + \mathbf{w}_k \quad (3)$$



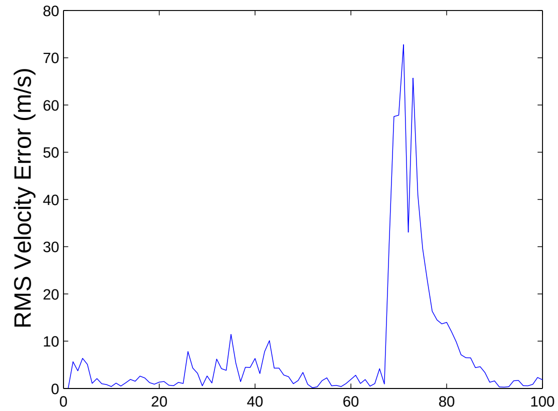
(a) CDKF



(b) CKF



(c) CDKF



(d) CKF

Fig. 1.

The interval between two consecutive measurements is assumed to be $T = 5$s. The measurement noise covariance $\mathbf{R}$ is given by

$$\mathbf{R} = \text{cov}[\mathbf{w}_k] \quad = \quad \text{diag}([\sigma_r^2 \ \sigma_\theta^2]).$$